Probability and Statistics

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Probability and Statistics:

Standard 1 — Descriptive Statistics

Students create, compare, and evaluate data displays using such methods as histograms, cumulative distribution functions, and scatterplots. For these data, they calculate measures of central tendency (various kinds of mean, the median, and the mode) and their derivatives (range, quartiles, variance, and standard deviation).

Standard 2 — Probability

Students understand the counting principle, permutations, and combinations and use them to solve problems. They develop rules for finding probabilities of combined and complementary events. They understand and use conditional probability and the related Bayes' Theorem. They investigate probability distributions and calculate and interpret their means and variances. They use and apply the normal distribution, including using the central limit theorem.

Standard 3 — Statistical Inference

Students use confidence intervals and hypothesis tests of means and differences between means. They use the principle of least squares to find curves of best fit and they calculate and interpret correlation coefficients.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to investigating probability situations and applying them to distributions, confidence intervals, and hypothesis tests.

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical ideas includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Descriptive Statistics

Students gather and display data and use measures of central tendency and variability.

PS.1.1 Create, compare, and evaluate different graphic displays of the same data, using histograms, frequency polygons, cumulative frequency distribution functions, pie charts, scatterplots, stem-and-leaf plots, and box-and-whisker plots. Draw these by hand or use a computer spreadsheet program.

Example: Gather data to answer the question: Which area of the country has the highest high school dropout rate? Display your dropout data in various forms.

PS.1.2 Compute and use mean, median, mode, weighted mean, geometric mean, harmonic mean, range, quartiles, variance, and standard deviation.

Example: Use spreadsheet formulas to compute measures that summarize your dropout data by state.

Standard 2

Probability

Students solve problems involving the use of probability and probability distributions.

PS.2.1 Understand the counting principle, permutations, and combinations and use them to solve problems.

Example: A chess team has 5 players available. How many different teams of 3 could you have? If you are one of the players, what is the probability that you will be chosen when a team is selected at random? Show that the number of different teams of 2 players is the same as the number with 3 players. Explain why this is true.

PS.2.2 Understand and use the addition rule to calculate probabilities for mutually exclusive and nonmutually exclusive events.

Example: When you roll 3 dice, find the probability of obtaining an even total or a total less than 7.

PS.2.3 Understand and use the multiplication rule to calculate probabilities for independent and dependent events.

Example: When you roll 3 dice, find the probability of obtaining a 6, followed by an even number, followed by a 4.

PS.2.4 Calculate the probabilities of complementary events.

Example: Find the probability that at least two people in a group of 10 will have the same birthday.

PS.2.5 Understand conditional probability and Bayes' Theorem and use them to solve problems.

Example: The test for a disease is positive for 100% of those with the virus, but one person in 20,000 without the virus also tests positive. If one person in 1,000 actually has the virus, what is the probability that a person who tests positive does have the virus?

PS.2.6 Use discrete random variables and probability distributions, including the binomial and geometric distributions.

Example: When you flip a coin 5 times, the number of heads is 0, 1, 2, 3, 4, or 5. Find the probability of each number of heads and draw a histogram of the results.

PS.2.7 Compute and interpret the mean and variance of a probability distribution.

Example: In the last example, find the mean and variance of the number of heads.

PS.2.8 Use and apply the normal distribution.

Example: Math SAT scores are normally distributed with mean 500 and standard deviation 100. What is the probability that Joan's SAT score is greater than 550?

PS.2.9 Understand the central limit theorem and use it to solve problems.

Example: Compare the means of small samples of the lengths of words used in different sections of a newspaper (e.g., editorial, news, sports).

PS.2.10 Use other continuous random variables and probability distributions to solve problems.

Example: Compare graphs of the normal distribution with binomial, uniform, linear, and exponential distributions. Draw sets of graphs with a computer or graphing calculator.

Standard 3

Statistical Inference

Students use confidence intervals and hypothesis tests, fit curves to data, and calculate correlation coefficients.

PS.3.1 Compute and use confidence intervals to make estimates.

Example: Using the mean and standard deviation of a sample, calculate 95% confidence intervals for the true mean (assuming normality).

PS.3.2 Understand hypothesis tests of means and differences between means and use them to reach conclusions.

Example: Test a hypothetical mean using the mean and standard deviation of a sample (assuming normality).

PS.3.3 Use the principle of least squares to find the curve of best fit for a set of data.

Example: Measure the wrist and neck size of each person in your class and make a scatterplot. Find the median fit line and the least squares regression line. Which line is a better fit? Explain your reasoning.

PS.3.4 Calculate and interpret the correlation coefficient of a set of data.

Example: Calculate and interpret the correlation coefficient for the linear regression model in the last example. Graph the residuals and evaluate the fit of the linear equation.

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